

# Advanced Data Structures and Algorithms

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*Learn how to enhance data processing with more complex and advanced data structures*

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Abirami A  
Priya R L



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## Preface

This book covers a collection of complex algorithms and helps to face the challenges in algorithmic analysis. Analysis of algorithms and handling sophisticated data structures focus on the fundamentals of the computer programming field. The book highlights how to find the best optimal solution to a real-world problem using an appropriate algorithm. The book provides theoretical explanations and solved examples of most of the topics covered.

This book also introduces the importance of performance analysis of an algorithm, which helps to increase efficiency and reduces time and space complexity. It shows how to create and design a complex data structure. This book solves the basic understanding of greedy and dynamic programming. It also gives importance to various divide-and-conquer techniques. This book gives information about string-matching methods as well.

This book is divided into six chapters. The reader will go through advanced data structures, greedy and dynamic programming, optimal solutions, string matching using various techniques, and calculations of time and space complexity using Asymptotic notations. To help learners better comprehend the material, each topic is handled with appropriate examples. The specifics are mentioned as follows.

**Chapter 1** emphasizes the basics of algorithmic analysis. It will discuss the need for analysis of algorithms and help us to choose a better suitable algorithm for a given problem statement. In algorithmic design, the complexity of an algorithm plays an important aspect to justify the design decisions. Accordingly, algorithm efficiency is measured from two perspectives such as time and space complexity. Hence, the major focus of this chapter is on various types of asymptotic notations used for the estimation of the time complexity of an algorithm and is discussed with examples.

**Chapter 2** discusses different complex data structures that may be used to effectively tackle difficult situations. AVL Tree, Huffman Coding, Redblack Tree, and several more search trees are among the advanced data structures addressed.

**Chapter 3** discusses the divide and conquer technique with the basic introduction and various methods that are involved in it with suitable examples. Divide and Conquer is the simplest and easiest technique of decomposing a larger problem into simpler problems, to solve any given problem statement.

**Chapter 4** will cover information about various greedy algorithms such as the knapsack problem, optimal merge pattern, subset cover problem, and so on, in detail with various solved examples.

**Chapter 5** discusses Dynamic Programming. It describes various classical computer science problems and their optimal solutions using dynamic programming approaches along with their applications. The need for dynamic algorithms and introduction to NP-Hard & NP-Complete are discussed with examples.

**Chapter 6** describes different string-matching algorithms with suitable examples. The chapter also provides description of genetic algorithms.

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# CHAPTER 1

# Analysis of Algorithm

## Introduction

The chapter emphasizes the basics of algorithmic analysis. *Donald Knuth* defines the term “*analysis of algorithms*” as the common technique for theoretical estimation of required resources, that is used to provide a solution to any specific computational problem. It will discuss the need for analysis of algorithms and help us choose a more suitable algorithm for a given problem statement. In algorithmic design, complexity of an algorithm plays an important aspect in justifying the design decisions. Accordingly, algorithm efficiency is measured in two perspectives, such as time and space complexity. Hence, the major focus of this chapter is on various types of asymptotic notations used for the estimation of time complexity of an algorithm and is discussed with examples.

## Structure

In this chapter, we will discuss the following topics:

- Analysis of algorithm
- Asymptotic Notations
- Time Complexity
- General Rules for time complexity calculation
- Recurrences

## Objectives

This chapter discusses the basics of analysis of algorithm, the need for analysis of algorithms as well as the various notations, with examples. Apart from these, time complexity calculation is the major content focused on, in the chapter.

## Analysis of algorithm

In this section, we give an overview of a generic framework on analysis of algorithms. It analyzes the efficiency of algorithms, in terms of space efficiency and time efficiency. To represent the complexity of an algorithm, asymptotic notations are a very crucial and important factor in designing algorithms. Therefore, various notations with solved examples are illustrated here.

Space complexity is defined as the amount of memory space required to execute an algorithm. Sometimes, space complexity is ignored as the space used is minimal. But time complexity refers to the amount of time required for the computation of an algorithm. Mostly, execution time depends on the various properties such as disk input/output speed, CPU speed, instructor set and so on. The calculation of time complexity and its rules with solved examples are also described in this chapter. It is followed by the recurrences in algorithm analysis and its three major types are discussed in the later sections of this chapter.



## What is analysis of algorithms?

In general terms, analysis of algorithms discusses the efficiency of an algorithm. It tells the estimation of various resources required by an algorithm to crack a specific problem of computation. The resources are the necessary storage or the required time to execute a specific algorithm. The estimated running time of an algorithm is called time complexity and the estimated storage/memory needed for the execution of an algorithm is called space complexity.

## Why to analyze algorithms?

Multiple solutions are available for a single problem; analysis will present the best algorithm to solve the given problem out of the multiple solutions. Even though the objective of the algorithms is to generate the expected output, the ways in which the outputs are generated, are different. The algorithm varies in the time and the space complexity. The various cases of analysis such as the worst, best and the average are performed with the help of asymptotic notations, to finalize the best algorithm.

## Asymptotic notations

The main idea of asymptotic analysis is to have a measure of efficiency of algorithms, that does not depend on machine specific constants, and neither requires algorithms to be implemented and time taken by programs to be compared. Asymptotic notations are mathematical tools to represent time complexity of algorithms for asymptotic analysis. The following 3 asymptotic notations are mostly used to represent time complexity of algorithms.

### Θ Notation

In this *theta* notation, the function bounds between the upper and the lower bound, and so it is called as an average case of  $T(n)$ .

*Average-case  $T(n)$  = average expected time of algorithm over all inputs of size  $n$ .*

$$\Theta(g(n)) = \left\{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ s.t.} \right. \\ \left. c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \right\}$$

Figure 1.1 features the average case Theta  $\Theta$  Notation:

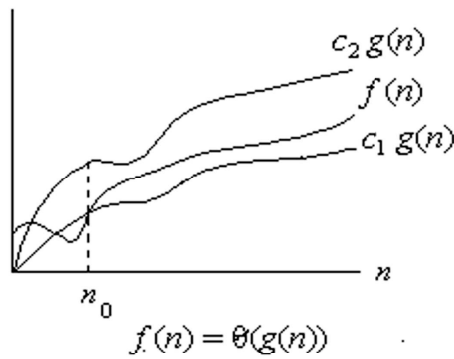


Figure 1.1: Average case Theta  $\Theta$  Notation

## Big O Notation

In this Big O notation, the function defines an upper bound, and so it is called the worst case of  $T(n)$ .

$T(n)$  = maximum time of algorithm on any input of size  $n$ .

$$O(g(n)) = \left\{ \begin{array}{l} f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ s.t.} \\ f(n) \leq cg(n) \text{ for all } n \geq n_0 \end{array} \right\}$$

Figure 1.2 features the worst-case Big O Notation:

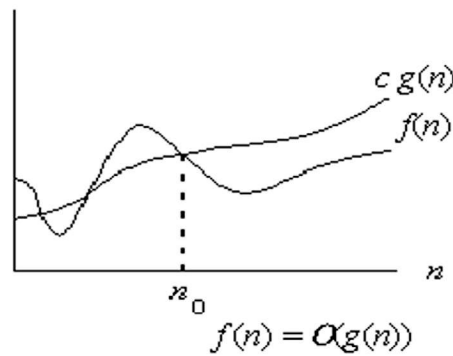


Figure 1.2: Worst case Big O Notation

## Ω (Omega) Notation

In this Ω notation, the function defines a lower bound, and so it is called the best case of T(n).

$T(n)$  = minimum time of algorithm on any input of size  $n$ .

It is a slow algorithm that works fast on some input.

$$\Omega(g(n)) = \left\{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ s.t.} \right. \\ \left. cg(n) \leq f(n) \text{ for all } n \geq n_0 \right\}$$

Figure 1.3 features the best-case Omega Ω Notation:

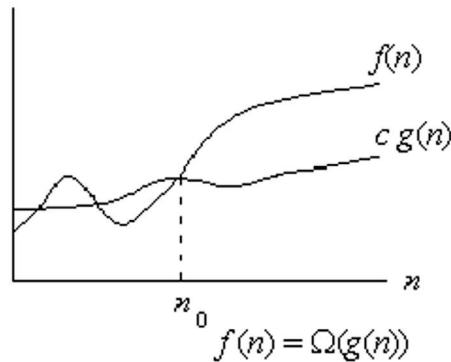


Figure 1.3: Best case Omega Ω Notation

Table 1.1 features the Asymptotic notations:

Asymptotic Notations	Case	Function Bound
Big O	Worst Case	Upper bound
Omega Ω	Best Case	Lower Bound
Theta Θ	Average Case	Tight Bound

Table 1.1: Asymptotic Notations

### Example 1

Find Upper Bound, Lower Bound and Tight Bound for the following function  $2n+5$ .

**Solution 1:**

Consider  $f(n)=2n+5$ ,

$$g(n)=n \text{ \{consider the highest degree of } f(n)\}$$

Refer to Table 1.2:

Lower Bound	Tight Bound	Upper Bound
Omega $\Omega$	Theta $\Theta$	Big O
Best Case	Average case	Worst case
$2n$	$2n$	$3n$

*Table 1.2: The constant value consideration for the asymptotic notations*

**Big O:**

$$f(n) \leq c * g(n)$$

$$2n+5 \leq c * n$$

$$C=3 // \text{ as per Table 1.2}$$

$$2n+5 \leq 3 * n$$

$$\text{For } n=1, 7 \leq 3 \rightarrow \text{False}$$

$$\text{For } n=2, 9 \leq 6 \rightarrow \text{False}$$

$$\text{For } n=3, 11 \leq 9 \rightarrow \text{False}$$

$$\text{For } n=4, 13 \leq 12 \rightarrow \text{False}$$

$$\text{For } n=5, 15 \leq 15 \rightarrow \text{True}$$

$$\text{Therefore } f(n)=O(g(n))$$

$$2n+5=O(n) \text{ for all } n \geq 5, C=3$$

**Omega ( $\Omega$ ):**

$$f(n) \geq c * g(n)$$

$$2n+5 \geq c * n$$

$$C=2$$

$$2n+5 \geq 2 * n$$

$$\text{For } n=1, 7 \geq 2 \rightarrow \text{True}$$

Therefore  $f(n) = \Omega(g(n))$

$2n+5 = \Omega(n)$  for all  $n \geq 1$ ,  $C=2$

**Theta ( $\Theta$ ):**

$C1 * g(n) \leq f(n) \leq c2 * g(n)$

$C1 * n \leq 2n+5 \leq c2 * n$

$C1=2, c2=3$

$2 * n \leq 2n+5 \leq 3 * n$

For  $n=1, 2 \leq 7 \leq 3 \rightarrow \text{false}$

For  $n=2, 4 \leq 9 \leq 6 \rightarrow \text{False}$

For  $n=3, 6 \leq 11 \leq 9 \rightarrow \text{false}$

For  $n=4, 8 \leq 13 \leq 12 \rightarrow \text{false}$

For  $n=5, 10 \leq 15 \leq 15 \rightarrow \text{true}$

Therefore  $f(n) = \Theta(g(n))$

$2n+5 = \Theta(n)$  for all  $n \geq 5$ ,  $c1=2, c2=3$

## Example 2

Find Upper Bound, Lower Bound and Tight Bound for the following function  $3n+2$

**Solution 2:**

**Big O:**

$f(n) = O(g(n));$

$3n+2 = O(n)$  for all  $n \geq 2$ ,  $c=4$

**Omega  $\Omega$ :**

$F(n) = \Omega(g(n));$

$3n+2 = \Omega(n)$  for all  $n \geq 1$ ,  $c=3$

**Theta  $\Theta$ :**

$F(n) = \Theta(g(n));$

$3n+2 = \Theta(n)$  for all  $n \geq 2$ ,  $c1=3$  &  $c2=4$ .